

## 2 Making Models with Polynomials

$$[1, 2, 3]$$

↳ vector space  $a, b$

$$a + b \quad \text{number} \cdot a$$

assoc:  $(a + b) + c = a + (b + c),$

$$(\alpha + \beta) \cdot c = \alpha \cdot c + \beta \cdot c,$$

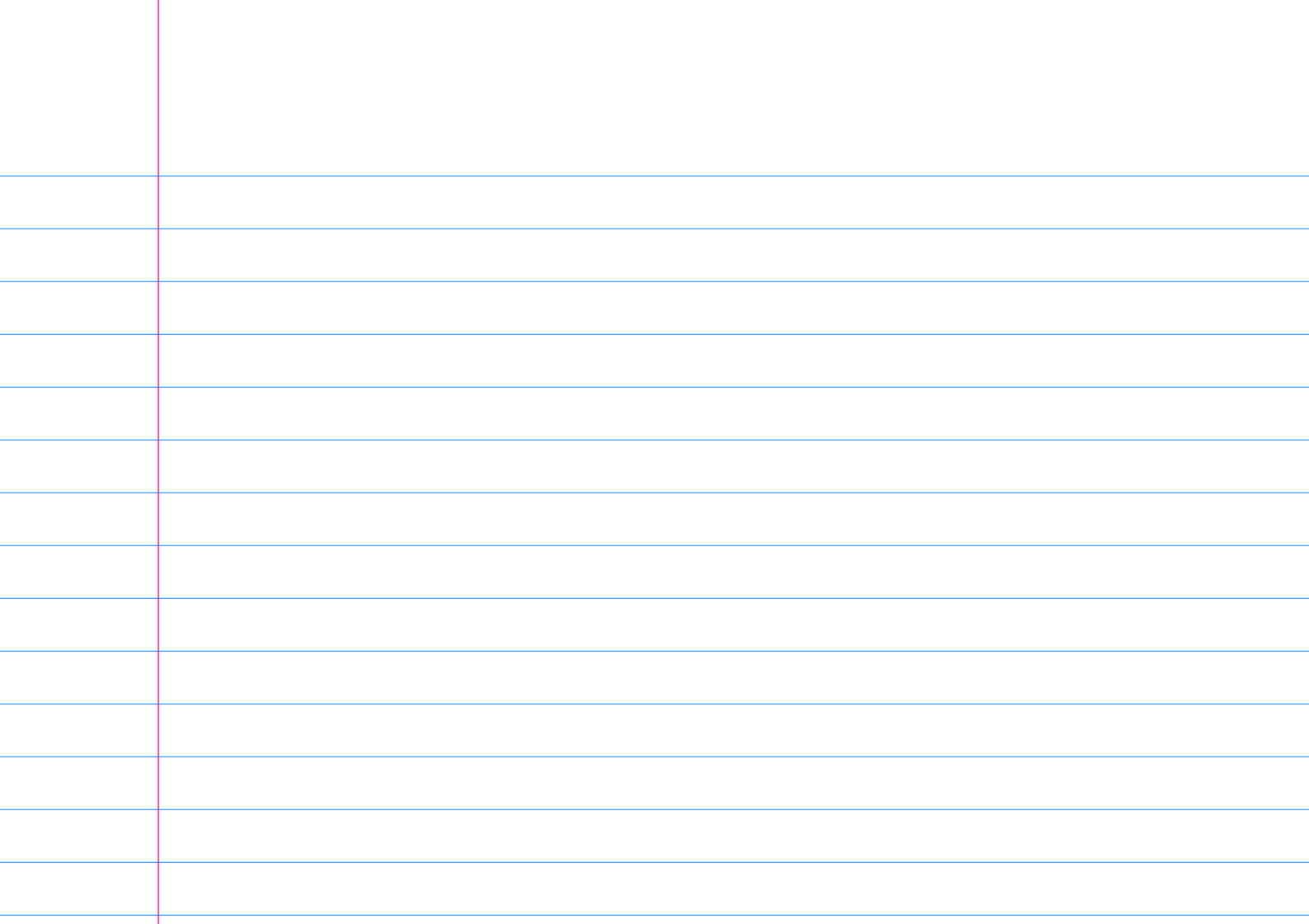
18 multiple choice Q  $\rightarrow$  7 pt

3 coding Q  $\rightarrow$  7 pt

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coding, LA, poly, Taylor, big O <sup>24 pts / 22 pts</sup>

$\hookrightarrow$  Python, numpy docs available



## Why polynomials?

$$a_3x^3 + a_2x^2 + a_1x + a_0$$

- How do we write the general case?
- *Why* polynomials and not something else?

Add + Multiply

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## Reconstructing a Function From Derivatives

- If we know  $f(x_0), f'(x_0), f''(x_0)$ , can we reconstruct the function as a polynomial?

$$f(x) = ??? + ???x + ???x^2 + \dots$$

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(0)}{i!} x^i$$

$$= f(0) + \frac{f'(0)}{1} x + \dots$$

## **Demo:** Polynomial Approximation with Polynomials (Part I)

## Shifting the Expansion Center

- Can you do this at points other than the origin?

$$f(x) = \sum_{i=0}^n \frac{f^{(i)}(0)}{i!} x^i \quad \left\{ \begin{array}{l} 0 \mapsto x_0 \\ x \mapsto x - x_0 \end{array} \right.$$
$$f(x_0 + x - x_0) = \sum_{i=0}^n \frac{f^{(i)}(x_0)}{i!} (x - x_0)^i$$
$$x - x_0 \mapsto h : f(x_0 + h) = \sum_{i=0}^n \frac{f^{(i)}(x_0)}{i!} h^i$$

## Errors in Taylor Approximation (I)

- Can't sum infinitely many terms. Have to **truncate**. How big of an error does this cause?

### Demo: Polynomial Approximation with Polynomials (Part II)

A Taylor expansion up to degree  $n$   
has error  $O(h^{n+1})$ .

$$\left| f(x) - \sum_{i=0}^n \frac{f^{(i)}(0)}{i!} x^i \right| = O(h^{n+1})$$

## Making Predictions with Taylor Truncation Error

- Suppose you expand  $\sqrt{x-10}$  in a Taylor polynomial of degree 3 about the center  $x_0=12$ . For  $h_1=0.5$ , you find that the Taylor truncation error is about  $10^{-4}$ .

What is the Taylor truncation error for  $h_2=0.25$ ?

$$10^{-4} = \text{Error}(h_1) = O(h_1^4) \approx C \cdot h_1^4$$

$$\text{Error}(h_2) = O(h_2^4) \approx C \cdot h_2^4$$

$$\begin{aligned} \text{Error}(h_2) &= C \cdot h_2^4 = C \cdot \cancel{h_1^4} \cdot \left(\frac{h_2}{h_1}\right)^4 \\ &= \text{Error}(h_1) \cdot \left(\frac{h_2}{h_1}\right)^4 \end{aligned}$$

## **Demo:** Polynomial Approximation with Polynomials (Part III)